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Spinor Strong Interaction Model for Baryon Spectra

F. C. Hoh[1]

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The spinor strong interaction model recently proposed by the author to account for meson spectra is applied to baryons. Quark–quark strong interaction is of massless scalar type. Harmonic confinement arises as naturally as linear confinement for mesons. No approximation is needed in order to derive, from the proposed covariant spinor baryon equations, coupled nonlinear radial equations for the ground-state spin-1/2 and spin-3/2 baryons in the rest frame. These equations are effectively of sixth order and call for a particle classification other than the usual unrelativistic one. Simplified analytical solutions are given. Internal functions and mass operators are analogously introduced. With these and the above simplified space-time solution, baryon data yield bare quark masses that agree approximately with those analogously obtained earlier from meson data.

1. INTRODUCTION

In the mid-1970-s, quantum chromodynamics (QCD) was introduced as the theory for strong interactions. Based in principle upon QCD, De Rujula *et al.* (1975) proposed a model to account for hadron spectra. This and similar approaches have since become the main ones pursued and improved upon by many authors, notably Isgur and Mitra, and have led to a sizable literature, part of which has been reviewed by Lichtenberg (1987).

These QCD-oriented models are basically phenomenological, making use of semirelativistic Hamiltonians. Since QCD has not been proven to offer confinement, confinement potentials are introduced ad hoc into the Hamiltonians to fit data. Generally, a harmonic type of confinement potential introduced earlier (Feynman *et al.*, 1971) for baryons and a linear type of confinement for mesons were adopted at large quark separations. At small quark distances, some approximate vector interaction among the

[1]Dragarbrunnsg. 9B, Uppsala, Sweden.

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quarks was assumed. By adjusting the parameters entering these models, good agreements with hadron spectra were obtained. The predictions, however, vary in accuracy, the parameters and their values differ, and a large number of predicted states are not seen.

This state of low-energy hadron theory is clearly unsatisfactory at least in regard to mathematical aesthetics and rigor when compared to the low-energy ends of classical gravitation and classical and quantum electrodynamics. If low-energy hadron theory is to rise to such levels, it is desirable that harmonic confinement for baryons and linear confinement for mesons arise naturally from a single unified theory which in addition can lead to predictions without loss of relativistic effects.

The purpose of this paper together with a recent one (Hoh, 1993; hereafter referred to as I) is to present such a unified theory and its consequences.

In I, a pseudoscalar interaction between the quarks replaces both the linear confinement potential and the vector interaction among the quarks employed in QCD-oriented models (Lichtenberg, 1987). The method of construction of the basic covariant meson equations departs from those of conventional relativistic quantum mechanics. Linear confinement arises naturally and radial equations for the ground-state mesons are derived keeping the relativistic effects intact. The theory leads to forms of predictions in agreement with the gross structure of meson spectra. It also predicts the masses of a number of pseudoscalar mesons within experimental error not predicted earlier (Hoh, n.d.-*a*).

Gauge invariance of these meson equations (Hoh, 1994) further predicts that Higgs particles are superfluous and naturally resolves the $U(1)$ problem. It also relates the W and Z gauge boson masses to the linear confinement constant. The present formalism thus provides a natural link between the strong and electroweak interactions different from that of the so-called grand unified theories.

In this paper, the spinor baryon equations are constructed in a way analogous to that leading to the spinor meson equations in Sec. 4 of I with the difference that a strong massless scalar quark–quark interaction replaces the strong pseudoscalar quark–antiquark interaction of I. In the rest frame, these equations are separable into a doublet ($S = 1/2$) and a quartet ($S = 3/2$) set of equations. These sets can be reduced to equations in 3 dimensional relative space and yield harmonic confinement without approximation. For the ground state spin 1/2 and 3/2 baryons, these equations further reduce to two coupled third order nonlinear integro-differential eigenvalue problems in one dimension. Simplified analytical solutions to these equations are given. This part is treated in Sec. 2 and 5–7 below.

The internal part of this paper is treated in Sec. 3 and 8 which are equivalent to Sec. 5 and 9 of I. Internal functions and mass operators for baryons are introduced and a simple model for these is proposed. Sec. 4 gives a selection rule for the combined internal and space time baryon wave functions.

In Sec. 9, application of this theory is given and discussed for 3 levels of increasing degree of contact with data.

2. CONSTRUCTION OF SPINOR BARYON EQUATIONS

In the following, the symbols and their definitions, unless stated otherwise, will be the same as those in I or obvious. Equations in I will be referred to by the same number preceded by a I.

The quark and antiquark of a meson in I were assumed to act upon each other via a massless pseudoscalar interaction. If a scalar interaction were employed, it will lead to a sign change of ϕ_P in (I4.9), so that the pseudoscalar mesons will not be confined, but the ground-state scalar mesons will, contrary to data. The three quarks in a baryon are, however, assumed to act upon each other via massless scalar interaction; pseudoscalar interaction is ruled out by parity invariance of the baryon equations (2.9) and (2.10) below.

Let x_{I}, x_{II}, and x_{III} denote the coordinates of the three quarks A, B, and C, respectively. For simplicity, x_{I}, x_{II}, and x_{III} as arguments are denoted by I, II, and III, respectively. The starting equations for the three quarks and their interactions corresponding to (I4.5)–(I4.7) are put in the form

$$\partial_{\text{I}}{}^{a\dot{b}}\chi_{A\dot{b}}(\text{I}) = i(m_A + V_{AB}(\text{I}) + V_{AC}(\text{I}))\psi^{a}_{A}(\text{I}) \tag{2.1a}$$

$$\partial_{\text{I}\dot{b}c}\psi^{c}_{A}(\text{I}) = i(m_A + V_{AB}(\text{I}) + V_{AC}(\text{I}))\chi_{A\dot{b}}(\text{I}) \tag{2.1b}$$

$$\partial_{\text{II}}{}^{d\dot{e}}\chi_{B\dot{e}}(\text{II}) = i(m_B + V_{BC}(\text{II}) + V_{BA}(\text{II}))\psi^{d}_{B}(\text{II}) \tag{2.2a}$$

$$\partial_{\text{II}\dot{e}f}\psi^{f}_{B}(\text{II}) = i(m_B + V_{BC}(\text{II}) + V_{BA}(\text{II}))\chi_{B\dot{e}}(\text{II}) \tag{2.2b}$$

$$\partial^{g\dot{h}}_{\text{III}}\chi_{C\dot{h}}(\text{III}) = i(m_C + V_{CA}(\text{III}) + V_{CB}(\text{III}))\psi^{g}_{C}(\text{III}) \tag{2.3a}$$

$$\partial_{\text{III}\dot{h}k}\psi^{k}_{C}(\text{III}) = i(m_C + V_{CA}(\text{III}) + V_{CB}(\text{III}))\chi_{C\dot{h}}(\text{III}) \tag{2.3b}$$

$$\begin{aligned}\Box_{\text{I}}(V_{AB}(\text{I}) + V_{AC}(\text{I})) = {} & \frac{1}{2}g_A g_B(\psi^{e}_{B}(\text{I})\chi_{Be}(\text{I}) + \psi^{\dot{e}}_{B}(\text{I})\chi_{B\dot{e}}(\text{I})) \\ & + \frac{1}{2}g_A g_C(\psi^{g}_{C}(\text{I})\chi_{Cg}(\text{I}) + \psi^{\dot{g}}_{C}(\text{I})\chi_{C\dot{g}}(\text{I}))\end{aligned} \tag{2.4a}$$

$$\Box_{\rm II}(V_{BC}({\rm II}) + V_{BA}({\rm II})) = \frac{1}{2} g_B g_C (\psi^g_C({\rm II})\chi_{Cg}({\rm II}) + \psi^{\dot g}_C({\rm II})\chi_{C\dot g}({\rm II}))$$

$$+ \frac{1}{2} g_B g_A (\psi^a_A({\rm II})\chi_{Aa}({\rm II}) + \psi^{\dot a}_A({\rm II})\chi_{A\dot a}({\rm II})) \tag{2.4b}$$

$$\Box_{\rm III}(V_{CA}({\rm III}) + V_{CB}({\rm III})) = \frac{1}{2} g_C g_A (\psi^a_A({\rm III})\chi_{Aa}({\rm III}) + \psi^{\dot a}_A({\rm III})\chi_{A\dot a}({\rm III}))$$

$$+ \frac{1}{2} g_C g_B (\psi^e_B({\rm III})\chi_{Be}({\rm III}) + \psi^{\dot e}_B({\rm III})\chi_{B\dot e}({\rm III})) \tag{2.4c}$$

The genuine three-body interaction is dropped, as it will not appear from (2.8) on.

Following the procedure of Section 4 of I, we multiply the right and left sides of (2.1)–(2.3) together. The same is done to those of (2.4). Consider the product wave function obtained by multiplying together (2.1a), (2.3a), and (2.2b) and let it be generalized according to

$$\chi_{A\dot b}({\rm I})\chi_{C\dot h}({\rm III})\psi^f_B({\rm II}) \to \chi_{\dot b \dot h}{}^f({\rm I\ III\ II})$$

which is to be associated with baryons (Belifante, 1953). This spinor has eight components decomposable into a symmetric quartet and two mixed symmetry doublets. However, the ground-state baryons consist only of one quartet, assigned to spin 3/2, and one mixed symmetry doublet, assigned to spin 1/2.

To remove the extraneous doublet, let the position $x_{\rm III}$ of quark C be merged into the position $x_{\rm I}$ of quark A, i.e., let $x_{\rm III} \to x_{\rm I}$ after the above multiplication and generalize as follows:

$$\chi_{A\dot b}({\rm I})\chi_{C\dot h}({\rm III})\psi^f_B({\rm II}) \to \chi_{A\dot b}({\rm I})\chi_{C\dot h}({\rm I})\psi^f_B({\rm II}) \to \chi_{\{\dot b \dot h\}}{}^f({\rm I\ II}) \tag{2.5a}$$

Let A and C change places in (2.1)–(2.4) and carry out the same multiplications and procedures to obtain an expression analogous to (2.5a). These expressions are equivalent since their physical content is unaltered by such an interchange of labels. Therefore, the middle expression of (2.5a) is symmetric with respect to interchange of the subscripts A and C. Inside a baryon, it is not possible to distinguish quark A from quark C or their spin orientations. Therefore, the right expression of (2.5a) is also symmetric under interchange of $\dot b$ and $\dot h$, as is denoted by the braces, and hence has only six components. The product wave function on the right side of the product equation is analogously generalized into $\psi^{\{ag\}}{}_{\dot{e}}(\mathrm{I\,II})$. The same procedure is carried out for the product equation obtained from (2.1b), (2.3b), and (2.2a).

The right side of (2.5a) can be decomposed into a doublet and a quartet, much like the decomposition of (I6.1b). They are

$$\chi_{0\mathrm{i}} = \frac{1}{2}(\chi_{\{\mathrm{i}\dot{2}\}}{}^{1} - \chi_{\{\mathrm{i}\dot{\mathrm{i}}\}}{}^{2}), \qquad \chi_{0\dot{2}} = \frac{1}{2}(\chi_{\{\dot{2}\dot{2}\}}{}^{1} - \chi_{\{\dot{2}\mathrm{i}\}}{}^{2}) \tag{2.6a}$$

$$\begin{gathered} \chi_{3/2} = \chi_{\{\mathrm{i}\mathrm{i}\}}{}^{1}, \qquad \chi_{-3/2} = -\chi_{\{\dot{2}\dot{2}\}}{}^{2}, \qquad \chi_{1/2} = -\frac{1}{\sqrt{3}}(\chi_{\{\dot{2}\mathrm{i}\}}{}^{1} + \chi_{\{\mathrm{i}\dot{2}\}}{}^{1} + \chi_{\{\mathrm{i}\mathrm{i}\}}{}^{2}) \\ \chi_{-1/2} = \frac{1}{\sqrt{3}}(\chi_{\{\mathrm{i}\dot{2}\}}{}^{2} + \chi_{\{\dot{2}\mathrm{i}\}}{}^{2} + \chi_{\{\dot{2}\dot{2}\}}{}^{1}) \end{gathered} \tag{2.6b}$$

Entirely analogous expressions can be defined for ψ^{1}_{0}, ψ^{2}_{0}, $\psi_{\pm 3/2}$, and $\psi_{\pm 1/2}$ by letting $\chi \to \psi$, lower dotted indices $\to$ upper undotted indices, and vice versa on the right sides of (2.6). Equation (2.6b) shows that the quartet is totally symmetric under the interchange of any pair of indices. Equation (2.6a) shows that the doublet is of mixed symmetry, being symmetric in the first two indices inside the braces, but antisymmetric when an upper and a lower index are interchanged.

As quarks A and C have merged into a diquark type of object described by one set of space-time coordinates, the interaction between them vanishes. This corresponds to putting V_{AC}, V_{CA}, and $g_A g_C$ to zero in (2.4). The product equation obtained from (2.4a)–(2.4c) now becomes

$$\begin{aligned} &\Box_{\mathrm{I}} \Box_{\mathrm{I}} \Box_{\mathrm{II}} V_{AB}(\mathrm{I}) V_{CB}(\mathrm{I})(V_{BC}(\mathrm{II}) + V_{BA}(\mathrm{II})) \\ &\quad = \frac{1}{8} g^{2}_{A} g_{C} g^{3}_{B} \\ &\qquad \times \{\chi_{B\dot{b}}(\mathrm{I})\chi_{B\dot{h}}(\mathrm{I})\psi^{a}_{A}(\mathrm{II})\psi^{\dot{b}}_{B}(\mathrm{I})\psi^{\dot{h}}_{B}(\mathrm{I})\chi_{Aa}(\mathrm{II}) \\ &\qquad + \chi_{B\dot{b}}(\mathrm{I})\chi_{B\dot{h}}(\mathrm{I})\chi_{A\dot{a}}(\mathrm{II})\psi^{\dot{b}}_{B}(\mathrm{I})\psi^{\dot{h}}_{B}(\mathrm{I})\psi^{\dot{a}}_{A}(\mathrm{II}) \\ &\qquad + \chi_{B\dot{b}}(\mathrm{I})\psi^{a}_{B}(\mathrm{I})\psi^{h}_{A}(\mathrm{II})\psi^{\dot{b}}_{B}(\mathrm{I})\chi_{Ba}(\mathrm{I})\chi_{Ah}(\mathrm{II}) \\ &\qquad + \chi_{B\dot{b}}(\mathrm{I})\psi^{h}_{B}(\mathrm{I})\chi_{A\dot{a}}(\mathrm{II})\psi^{\dot{b}}_{B}(\mathrm{I})\chi_{Bh}(\mathrm{I})\psi^{\dot{a}}_{A}(\mathrm{II}) + \text{c.c.}\} \\ &\qquad + \frac{1}{8} g_{A} g^{2}_{C} g^{3}_{B} \times \{\text{same terms with } A \to C\} \end{aligned} \tag{2.7}$$

where c.c. denotes complex conjunction. As there are only two coordinates, there can only be two-body but not genuine three-body interaction. Again, inside a baryon, one cannot distinguish quark A from quark B or their spin orientations. Analogous to the $A-C$ symmetry discussed above, an exchange of A or C with B in the middle term of (2.5a) can at most produce a phase change φ_{ph} on the right side of (2.5a),

$$\chi_{B\dot{b}}(\mathrm{I})\chi_{B\dot{h}}(\mathrm{I})\psi^{f}_{A}(\mathrm{II}) \to \chi_{\{\dot{b}\dot{h}\}}{}^{f}(\mathrm{I\ II})\exp(i\varphi_{\mathrm{ph}}) \tag{2.5b}$$

A similar relation with $A \to C$ holds. Equation (2.7) can now be generalized via (2.5) and similar relations together with

$$V_{AB}(\mathrm{I})V_{CB}(\mathrm{I})(V_{BC}(\mathrm{II}) + V_{BA}(\mathrm{II})) \to \phi_S(\mathrm{I\ II}) \tag{2.8}$$

analogous to (I4.8). In (2.7), the phase factor of (2.5b) is removed by complex conjugation. As a result, the terms in the both pairs of braces in (2.7) become equal.

By analogy to Section 4 of I, the generalizations (2.5) and (2.8) together with the corresponding generalizations for the internal functions and operators in (3.3) and (3.5) below constitute the basic hypotheses of the present model and mark the departure from conventional relativistic quantum mechanics.

The equations so generalized are put in the following form associated with baryons:

$$\partial^{a\dot{b}}_{\mathrm{I}}\,\partial^{g\dot{h}}_{\mathrm{I}}\,\chi_{\{\dot{b}\dot{h}\}}{}^{f}(\mathrm{I\ II})\partial_{\mathrm{II}\,f\dot{e}} = -i(M^3_b + \phi_S(\mathrm{I\ II}))\psi^{\{ag\}}{}_{\dot{e}}(\mathrm{I\ II}) \tag{2.9a}$$

$$\partial_{\mathrm{I}\dot{b}c}\,\partial_{\mathrm{I}\dot{h}k}\,\psi^{\{ck\}}{}_{\dot{e}}(\mathrm{I\ II})\partial^{\dot{e}d}_{\mathrm{II}} = -i(M^3_b + \phi_S(\mathrm{I\ II})\chi_{\{\dot{b}\dot{h}\}}{}^{d}(\mathrm{I\ II}) \tag{2.9b}$$

$$\Box_{\mathrm{I}}\Box_{\mathrm{I}}\Box_{\mathrm{II}}\phi_S(\mathrm{I\ II}) = \frac{1}{4}g^6_q(\chi_{\{\dot{b}\dot{h}\}}{}^{f}(\mathrm{I\ II})(\psi^{\{bh\}}{}_{\dot{f}}(\mathrm{I\ II}))^* + \mathrm{c.c.}\}$$

$$g^6_q = \frac{1}{2}(g_A + g_C)g_A g_C g^3_B \tag{2.10}$$

where M^3_b replaces $m_A m_B m_C$ according to (3.5) and (3.8) below. Terms proportional to mVV and mmV in the product equations obtained from (2.1)–(2.3) are dropped, as are the second, third, and fourth terms and their complex conjugates in both pairs of braces in (2.7). The justification is entirely analogous to that preceding (I4.11) and is given in Appendix A.

The $A-C$ symmetry above shows that $g_A = g_C$ in (2.10). Also, there appears to be no reason for the magnitude of the coupling constant to be different for quark–quark and quark–antiquark interactions. The cubic root of the second of (2.10) is then equal to the quark–antiquark coupling constant $g_A g_B$ in Section 4 of I.

Take the complex conjugate of (2.9b) and multiply it by $\psi^{\{bh\}}{}_{\dot{d}}$. Adding the resulting equation to (2.9a) multiplied by $\chi_{\{ag\}}{}^{\dot{e}}$, the complex conjugate of $\chi_{\{\dot{a}\dot{g}\}}{}^{e}$, to obtain

$$\chi_{\{ag\}}{}^{\dot{e}}\,\partial_{\mathrm{I}}{}^{a\dot{b}}\,\partial_{\mathrm{I}}{}^{g\dot{h}}\,\partial_{\mathrm{II}\dot{e}f}\chi_{\{\dot{b}\dot{h}\}}{}^{f}+\psi^{\{bh\}}{}_{\dot{d}}\,\partial_{\mathrm{I}b\dot{c}}\,\partial_{\mathrm{I}h\dot{k}}\,\partial_{\mathrm{II}}{}^{ed}\psi^{\{\dot{c}\dot{k}\}}{}_{e}=0 \qquad (2.11\mathrm{a})$$

where $\psi^{\{\dot{c}\dot{k}\}}{}_{e}$ is the complex conjugate of $\psi^{\{ck\}}{}_{\dot{e}}$. Interchanging (2.9a) and (2.9b) and also χ and ψ, we find that the above procedure leads to

$$\chi_{\{\dot{a}\dot{g}\}}{}^{e}\,\partial_{\mathrm{I}}{}^{\dot{a}b}\,\partial_{\mathrm{I}}{}^{\dot{g}h}\,\partial_{\mathrm{II}e\dot{f}}\chi_{\{bh\}}{}^{\dot{f}}+\psi^{\{\dot{b}\dot{h}\}}{}_{d}\,\partial_{\mathrm{I}\dot{b}c}\,\partial_{\mathrm{I}\dot{h}k}\,\partial_{\mathrm{II}}{}^{de}\psi^{\{ck\}}{}_{\dot{e}}=0 \qquad (2.11\mathrm{b})$$

which is the complex conjugate of (2.11a).

If (2.9) is replaced by a Dirac equation in analogous spinor form, the corresponding sum of (2.11a) and (2.11b) becomes the continuity equation for the probability current density associated with the Dirac particle and leads to a conserved total probability. For a similar purpose, (2.11a) is chosen to be put in the form

$$\begin{aligned} D_{ad} \equiv\ & \partial_{\mathrm{I}}{}^{a\dot{b}}\chi_{\{ag\}}{}^{\dot{e}}\,\partial_{\mathrm{I}}{}^{g\dot{h}}\,\partial_{\mathrm{II}\dot{e}f}\chi_{\{\dot{b}\dot{h}\}}{}^{f}+\partial_{\mathrm{I}b\dot{c}}\psi^{\{bh\}}{}_{\dot{d}}\,\partial_{\mathrm{I}h\dot{k}}\,\partial_{\mathrm{II}}{}^{ed}\psi^{\{\dot{c}\dot{k}\}}{}_{e} \\ & +2\partial_{\mathrm{II}\dot{e}f}\chi_{\{ag\}}{}^{\dot{e}}\,\partial_{\mathrm{I}}{}^{a\dot{b}}\,\partial_{\mathrm{I}}{}^{g\dot{h}}\chi_{\{\dot{b}\dot{h}\}}{}^{f}+2\partial_{\mathrm{II}}{}^{ed}\psi^{\{bh\}}{}_{\dot{d}}\,\partial_{\mathrm{I}b\dot{c}}\,\partial_{\mathrm{I}h\dot{k}}\psi^{\{\dot{c}\dot{k}\}}{}_{e}=S_{ad} \qquad (2.12\mathrm{a}) \\ S_{ad} \equiv\ & (\partial_{\mathrm{I}}{}^{a\dot{b}}\chi_{\{ag\}}{}^{\dot{e}})(\partial_{\mathrm{I}}{}^{g\dot{h}}\,\partial_{\mathrm{II}\dot{e}f}\chi_{\{\dot{b}\dot{h}\}}{}^{f})+(\partial_{\mathrm{I}b\dot{c}}\psi^{\{bh\}}{}_{\dot{d}})(\partial_{\mathrm{I}h\dot{k}}\,\partial_{\mathrm{II}}{}^{ed}\psi^{\{\dot{c}\dot{k}\}}{}_{e}) \\ & +2(\partial_{\mathrm{II}\dot{e}f}\chi_{\{ag\}}{}^{\dot{e}})(\partial_{\mathrm{I}}{}^{a\dot{b}}\,\partial_{\mathrm{I}}{}^{g\dot{h}}\chi_{\{\dot{b}\dot{h}\}}{}^{f})+2(\partial_{\mathrm{II}}{}^{ed}\psi^{\{bh\}}{}_{\dot{d}})(\partial_{\mathrm{I}b\dot{c}}\,\partial_{\mathrm{I}h\dot{k}}\psi^{\{\dot{c}\dot{k}\}}{}_{e}) \qquad (2.12\mathrm{b}) \end{aligned}$$

3. INCLUSION OF INTERNAL COORDINATES

To describe the internal properties of baryons, internal coordinates are introduced analogous to Section 5 of I. Following the procedures there, (2.1) is multiplied from the right by an internal function $\xi^{p}_{A}(z_{\mathrm{I}})$ and m_A is replaced by an internal operator $m_{A\mathrm{op}}(z_{\mathrm{I}}, \partial/\partial z_{\mathrm{I}})$. The resulting equation is of the form of (I5.1) and replaces (2.1). Similarly, (2.2) and (2.3) are multiplied by $\xi^{q}_{B}(z_{\mathrm{II}})$ and $\xi^{s}_{C}(z_{\mathrm{III}})$, and m_B and m_C are replaced by $m_{B\mathrm{op}}(z_{\mathrm{II}}, \partial/\partial z_{\mathrm{II}})$ and $m_{C\mathrm{op}}(z_{\mathrm{III}}, \partial/\partial z_{\mathrm{III}})$, respectively. These generalized equations are multiplied together as in Section 2. Following (I5.3a) and the relation preceding (2.5), we make the generalization

$$\xi^{p}_{A}(z_{\mathrm{I}})\xi^{s}_{C}(z_{\mathrm{III}})\xi^{q}_{B}(z_{\mathrm{II}}) \rightarrow \xi^{psq}(z_{\mathrm{I}}, z_{\mathrm{III}}, z_{\mathrm{II}}) \qquad (3.1)$$

Consider an SU_3 case; the familiar Clebsch–Gordan series reduction of (3.1) reads

$$\underline{3}\times\underline{3}\times\underline{3}=\underline{10}+\underline{8}+\underline{8}+\underline{1} \qquad (3.2)$$

The ground-state baryons in the SU_3 scheme consist of a symmetric decuplet $\underline{10}$ and a mixed symmetry octet $\underline{8}$ associated with the symmetric $J^P=(3/2)^+$ and mixed symmetry $(1/2)^+$ of Section 2, respectively. Therefore, the unobserved extraneous $\underset{\sim}{8}$ and antisymmetric $\underset{\sim}{1}$ of (3.2) need to be removed, like the extraneous doublet mentioned above (2.5a). This appears to be necessary; if the extraneous $\underset{\sim}{8}$ and doublet were to be associated with each other, there would be no space-time function to be associated with the $\underset{\sim}{1}$.

The removal is achieved by following (2.5a) analogously and letting $z_{\rm III}$ merge into $z_{\rm I}$:

$$\xi^p_A(z_{\rm I})\xi^s_C(z_{\rm III})\xi^q_B(z_{\rm II}) \to \xi^p_A(z_{\rm I})\xi^s_C(z_{\rm I})\xi^q_B(z_{\rm II}) \to \xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}) \tag{3.3}$$

A discussion of the $A-C$ symmetry below (2.5a) can be carried over to the internal case here, and the right side of (3.3) is now symmetric in the first two indices p and s, indicated by the braces. It therefore has the required number of $\underset{\sim}{6} \times \underset{\sim}{3} = \underset{\sim}{10} + \underset{\sim}{8}$ components. The octet $\underset{\sim}{8}$ can be extracted by applying the antisymmetric operator ε_{sqr} to the right side of (3.3), leaving behind a totally symmetric decuplet $\underset{\sim}{10}$ indicated by the braces in (3.4b),

$$\eta^p_r(z_{\rm I}, z_{\rm II}) = \xi^{\{ps\}q}(z_{\rm I}, z_{\rm II})\varepsilon_{sqr} \qquad \underset{\sim}{8} \tag{3.4a}$$

$$\xi^{\{psq\}}(z_{\rm I}, z_{\rm II}) \qquad \underset{\sim}{10} \tag{3.4b}$$

The $\underset{\sim}{8}$ of (3.4a) is of mixed symmetry, being symmetric in p and s in the braces, but antisymmetric under the interchange of the last two indices. The trace of η^p_r vanishes.

By analogy to (I5.3b) and (2.8), the product of the mass operators is similarly generalized:

$$\begin{aligned} m_A m_C m_B &\to m_{A{\rm op}}(z_{\rm I}, \partial/\partial z_{\rm I}) m_{C{\rm op}}(z_{\rm III}, \partial/\partial z_{\rm III}) m_{B{\rm op}}(z_{\rm II}, \partial/\partial z_{\rm II}) \\ &\to m_{A{\rm op}}(z_{\rm I}, \partial/\partial z_{\rm I}) m_{C{\rm op}}(z_{\rm I}, \partial/\partial z_{\rm I}) m_{B{\rm op}}(z_{\rm II}, \partial/\partial z_{\rm II}) \\ &\to m_{3{\rm op}}(z_{\rm I}, \partial/\partial z_{\rm I}, z_{\rm II}, \partial/\partial z_{\rm II}) \end{aligned} \tag{3.5}$$

With these generalizations, (2.9) becomes

$$\begin{aligned} &\partial_{\rm I}^{a\dot{b}}\, \partial_{\rm I}^{g\dot{h}} \chi_{\{\dot{b}\dot{h}\}}{}^f({\rm I\ II})\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II})\partial_{{\rm II}f\dot{e}} \\ &\quad = -i(m_{3{\rm op}}(z_{\rm I}, \partial/\partial z_{\rm I}, z_{\rm II}, \partial/\partial z_{\rm II}) + \phi_S({\rm I\ II}))\psi^{\{ag\}}{}_{\dot{e}}({\rm I\ II})\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}) \end{aligned} \tag{3.6a}$$

$$\begin{aligned} &\partial_{{\rm I}\dot{b}c}\, \partial_{{\rm I}\dot{h}k} \psi^{\{ck\}}{}_{\dot{e}}({\rm I\ II})\partial_{\rm II}{}^{\dot{e}d}\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}) \\ &\quad = -i(m_{3{\rm op}}(z_{\rm I}, \partial/\partial z_{\rm I}, z_{\rm II}, \partial/\partial z_{\rm II}) + \phi_S({\rm I\ II})\chi_{\{\dot{b}\dot{h}\}}{}^d({\rm I\ II})\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}) \end{aligned} \tag{3.6b}$$

The total baryon wave functions

$$\chi_{\{\dot{b}\dot{h}\}}{}^f({\rm I\ II})\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}), \qquad \psi^{\{ck\}}{}_{\dot{e}}({\rm I\ II})\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}) \tag{3.7}$$

must now be eigenfunctions of $m_{3{\rm op}}$ according to

$$m_{3{\rm op}}(z_{\rm I}, \partial/\partial z_{\rm I}, z_{\rm II}, \partial/\partial z_{\rm II})\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}) = M_b^3\xi^{\{ps\}q}(z_{\rm I}, z_{\rm II}) \tag{3.8}$$

where M_b^3 denotes its eigenvalue. Equations (3.6), (3.8), and (2.10) are the proposed spinor baryon equations in the present model, subjected to the symmetry condition (4.1) below.

4. SYMMETRIC HADRON WAVE FUNCTION POSTULATE

The total baryon wave functions of (3.7) associate the mixed symmetry doublet (2.6a) with the mixed symmetry $\underset{\sim}{8}$ of (3.4a) to represent the observed spin-1/2 octet baryons. Similarly, the totally symmetric (2.6b) and (3.4b) are combined to represent the observed spin-3/2 decuplet baryons. However, (3.7) also associates the same mixed symmetry doublet to the totally symmetric $\underset{\sim}{10}$ and the same totally symmetric quartet to the mixed symmetry $\underset{\sim}{8}$, contrary to data. To eliminate the representations of these unobserved states, the following postulate is made.

The total hadron wave function must be totally symmetric under simultaneous interchange of the space-time index and the internal index associated with a quark with those of another quark or antiquark.

The essence of this postulate is equivalent to that of the so-called symmetric quark model, which, for instance, leads to the approximately correct baryon magnetic moment relations (Lichtenberg, 1978).

Applied to baryons, the postulate means that (3.7) must be subjected to the condition

$$\begin{aligned} \chi_{\{\dot{b}\dot{h}\}}{}^{f}\xi^{\{ps\}q} &= \chi_{\{\dot{b}\dot{f}\}}{}^{h}\xi^{\{pq\}s} \\ \psi_{\{ck\}\dot{e}}\xi^{\{ps\}q} &= \psi^{\{ce\}}{}_{\dot{k}}\xi^{\{pq\}s} \end{aligned} \tag{4.1}$$

For mesons, the above postulate led to (I9.2). This postulate plays a role similar to that of Pauli's theorem in conventional quantum mechanics in eliminating extraneous solutions. The role of identical particles in Pauli's theorem is taken over here by quarks which are indistinguishable, hence appearing as identical, inside hadrons.

5. REDUCTION OF THE SPACE-TIME EQUATIONS

The space-time part of (3.6), i.e., (2.9) and (2.10), is now reduced following the procedure of Section 6 of I in the rest frame. Introduce the relative and laboratory frame coordinates

$$x^\mu = x_{\mathrm{II}}{}^\mu - x_{\mathrm{I}}{}^\mu, \qquad X^\mu = (1-a_b)x_{\mathrm{I}}{}^\mu + a_b x_{\mathrm{II}}{}^\mu$$

Further, let g_q^6 be absorbed into the ψ's and χ's analogous to (I6.11). Solutions of the following form are sought:

$$g_q^3\chi_{\{\dot{b}\dot{h}\}}{}^{f}(\mathrm{I\ II}) = e^{-iK_\mu X^\mu}\chi_{\{\dot{b}\dot{h}\}}{}^{f}(x^\mu) \tag{5.1a}$$

$$g_q^3\psi^{\{ck\}}{}_{\dot{e}}(\mathrm{I\ II}) = e^{-iK_\mu X^\mu}\psi^{\{ck\}}{}_{\dot{e}}(x^\mu) \tag{5.1b}$$

Here, $K_\mu = (E_0, -\mathbf{K})$. The relative time x^0 dependence is assumed to be of the form $\exp(i\omega_0 x^0)$ and the choice $a_b = \omega_0/E_0 + 1/2$ is made (Hoh, 1993). Here, ω_0 denotes the relative energy. Equation (2.9) in the rest frame $\mathbf{K} = 0$ now becomes

$$\begin{aligned}(i\delta^{a\dot{b}}E_0/2 - \partial^{a\dot{b}})(i\delta^{g\dot{h}}E_0/2 - \partial^{g\dot{h}})\chi_{\{\dot{b}\dot{h}\}}{}^{f}(\mathbf{x})(i\delta_{f\dot{e}}E_0/2 + \partial_{f\dot{e}})&\\ = -i(M_b^3 + \phi_S(\mathbf{x}))\psi^{\{ag\}}{}_{\dot{e}}(\mathbf{x})& \qquad (5.2\text{a})\end{aligned}$$

$$\begin{aligned}(i\delta^{\dot{b}c}E_0/2 - \partial^{\dot{b}c})(i\delta_{\dot{h}k}E_0/2 - \partial_{\dot{h}k})\psi^{\{ck\}}{}_{\dot{e}}(\mathbf{x})(i\delta^{\dot{e}d}E_0/2 + \partial^{\dot{e}d})&\\ = -i(M_b^3 + \phi_S(\mathbf{x}))\chi_{\{\dot{b}\dot{h}\}}{}^{d}(\mathbf{x})& \qquad (5.2\text{b})\end{aligned}$$

Here, $\delta^{a\dot{b}} \equiv \delta^a_{\dot{b}}$, and $\partial^{a\dot{b}}$ and $\partial_{f\dot{e}}$ refer to $\mathbf{x}$ in x^μ and are given by (I.A2) without the subscript I and with $\partial_0 \to 0$. $\phi_S(\mathbf{x})$ is given by a similar reduction of (2.10):

$$-\Delta\Delta\Delta\phi_S(\mathbf{x}) = \frac{1}{4}\left(\chi_{\{\dot{b}\dot{h}\}}{}^{f}(\mathbf{x})(\psi^{\{bh\}}{}_{f}(\mathbf{x}))^* + \text{c.c.}\right) \qquad (5.3)$$

By applying the ε operators in (I6.1a) to (5.1a), one can separate off a doublet of the form of (2.6a), whose ground state has spin 1/2, leaving behind a quartet of the form of (2.6b), whose ground state has spin 3/2. Similar operations can be carried out for (5.1b), (5.2), and (5.3). Putting the $S = 3/2$ components in (5.2) and (5.3) to 0, we obtain for the remaining $S = 1/2$ equations

$$(i\delta^{a\dot{b}}E_{0\mathrm{d}}/2 - \partial^{a\dot{b}})(E_{0\mathrm{d}}^2/4 + \Delta)\chi_{0\dot{b}}(\mathbf{x}) = -i(M_{b\mathrm{d}}^3 + \phi_{S\mathrm{d}}(\mathbf{x}))\psi_0{}^{a}(\mathbf{x}) \qquad (5.4\text{a})$$

$$(i\delta_{\dot{b}c}E_{0\mathrm{d}}/2 - \partial_{\dot{b}c})(E_{0\mathrm{d}}^2/4 + \Delta)\psi_0^c(\mathbf{x}) = -i(M_{b\mathrm{d}}^3 + \phi_{S\mathrm{d}}(\mathbf{x}))\chi_{0\dot{b}}(\mathbf{x}) \qquad (5.4\text{b})$$

$$\Delta\Delta\Delta\phi_{S\mathrm{d}}(\mathbf{x}) = \frac{4}{3}\,\mathrm{Re}\,\psi_0^a(\mathbf{x})\chi_{0\dot{a}}^*(\mathbf{x}) \qquad (5.5)$$

Here, the subscript d refers to specialization to doublets; the subscript q below refers to quartets.

The remaining equations in (5.2) with totally symmetric wave functions and $\phi_S \to \phi_{S\mathrm{q}}$ are assigned to $S = 3/2$. $\phi_{S\mathrm{q}}$ is obtained from (5.3) by putting the $S = 1/2$ functions ψ_0 and χ_0 to 0:

$$\Delta\Delta\Delta\phi_{S\mathrm{q}}(\mathbf{x}) = -\frac{1}{4}\left(\sum_{\nu=-3/2}^{3/2} \chi_\nu(\mathbf{x})\psi_\nu^*(\mathbf{x}) + \text{c.c.}\right) \qquad (5.6)$$

For $\mathbf{K} \neq 0$, the full (2.9) must be used. Analogous to what is said above (I6.12), the $S = 3/2$ components of (2.6b) will be of order $\mathbf{K}$ and appear as "small" components in (2.9) for a reduction to the $S = 1/2$ equations (5.4) when $\mathbf{K} = 0$. The roles of $S = 3/2$ and $S = 1/2$ components in (2.6) are switched if (2.9) reduces to the $S = 3/2$ part of (5.2) as $\mathbf{K} \to 0$.

6. HARMONIC CONFINEMENT AND THE $S = 1/2$ EQUATIONS

The Green's function for (5.5) and (5.6) is given by

$$\Delta\Delta\Delta G_b(\mathbf{x}, \mathbf{x}') = \delta(\mathbf{x} - \mathbf{x}') \tag{6.1a}$$

$$G_b(\mathbf{x}, \mathbf{x}') = -(1/96\pi)|\mathbf{x} - \mathbf{x}'|^3 \tag{6.1b}$$

There are also homogeneous solutions to (6.1a) proportional to $|\mathbf{x}|$, $|\mathbf{x}|^{-1}$, $|\mathbf{x}|^4$, $|\mathbf{x}|^2$, and a constant. The $|\mathbf{x}|^2$ term can be dropped according to the reasoning below (I7.1). It can also be shown that the $|\mathbf{x}|^4$ term belongs to the same category and can be neglected. Thus, (5.5) and (6.1) lead to

$$\phi_{Sd}(\mathbf{x}) = -\frac{1}{72\pi}\int d^3\mathbf{x}'|\mathbf{x} - \mathbf{x}'|^3 \operatorname{Re}(\psi_0^a(\mathbf{x}')\chi_{0\dot{a}}^*(\mathbf{x}')) + d_{11}|\mathbf{x}| + d_1 - d_{10}|\mathbf{x}|^{-1} \tag{6.2}$$

where the d's are integration constants.

Analogous to the observation preceding (I7.8), (5.4) and (6.2) assigned to the $S = 1/2$ baryons are also generally not separable in the r and ϑ space. In the $|\mathbf{x}| = r \to 0$ and ∞ limits, however, (6.2) shows that ϕ_{Sd} depends upon r only and separation is possible. Noting that (5.4) resembles Dirac's equations for a hydrogen atom but with the Coulomb potential replaced by a scalar potential, we can make the same form of wave function ansatz (Bethe and Salpeter, 1957):

$$\begin{aligned}\psi_0^a(\mathbf{x}) = {} & \mp g_0(r)\left(\frac{l \pm m + 1/2}{2l + 1}\right)^{1/2} Y_{lm\mp 1/2}(\vartheta, \varphi) \\ & + if_0(r)\left(\frac{l \mp m + 3/2}{2l + 3}\right)^{1/2} Y_{l+1m\mp 1/2}(\vartheta, \varphi)\end{aligned} \tag{6.3}$$

where the upper signs refer to $a = 1$ and the lower ones to $a = 2$. We obtain $\chi_{0\dot{a}}(\mathbf{x})$ by letting $g_0(r) \to -g_0(r)$ in (6.3). The radial equations associated with the Y_l and the Y_{l+1} components are

$$\begin{aligned}&\left[\frac{1}{2}E_{0d}\left(\frac{1}{4}E_{0d}^2 + \Delta_l\right) + M_{bd}^3 + \phi_{Sd}(\mathbf{x} \to 0, \infty)\right]g_0(r) \\ &\quad - \left(\frac{1}{4}E_{0d}^2 + \Delta_l\right)\left(\partial_r + \frac{l+2}{r}\right)f_0(r) = 0\end{aligned} \tag{6.4a}$$

$$\begin{aligned}&\left[\frac{1}{2}E_{0d}\left(\frac{1}{4}E_{0d}^2 + \Delta_{l+1}\right) - M_{bd}^3 - \phi_{Sd}(\mathbf{x} \to 0, \infty)\right]f_0(r) \\ &\quad + \left(\frac{1}{4}E_{0d}^2 + \Delta_{l+1}\right)\left(\partial_r - \frac{l}{r}\right)g_0(r) = 0\end{aligned} \tag{6.4b}$$

$$\partial_r = \partial/\partial r, \qquad \Delta_l = \partial_r^2 + \frac{2}{r}\partial_r - \frac{l(l+1)}{r^2} \tag{6.4c}$$

where

$$\phi_{Sd}(\mathbf{x}\to 0) = -d_{10}/r \tag{6.5a}$$

$$\phi_{Sd}(\mathbf{x}\to \pm) = -\frac{1}{72\pi} r^3 \int d^3x \operatorname{Re} \psi_0^a(\mathbf{x})\chi_{0\dot{a}}^*(\mathbf{x}) = \alpha_d^3 r^3 \tag{6.5b}$$

follow from (6.2).

One of the 6 asymptotic solutions to (6.4) and (6.5) is

$$g_0(r\to\infty) = -f_0(r\to\infty) = C_{d\infty} r^{\tau_d} e^{-\alpha_d r^2/2} \tag{6.6}$$

where $C_{d\infty}$ and τ_d are constants. A harmonic type of confinement thus arises as naturally as the linear confinement does in (I7.6a) and (I8.6b). It persists for $\alpha_d < 0$ if $f_0(r\to\infty)$ changes sign relative to $g_0(r\to\infty)$. Near the origin, the treatment follows closely that of Appendix B of I. There are six independent solutions:

$$g_0(r\to 0) = c_{g0}(\lambda_d) r^{\lambda_d}, \qquad \lambda_d = l+2, l, -l-1, \qquad f_0(r\to 0) = 0 \tag{6.7a}$$

$$f_0(r\to 0) = c_{f0}(\lambda_d) r^{\lambda_d}, \qquad \lambda_d = l+1, -l, -l-2, \quad g_0(r\to 0) = 0 \tag{6.7b}$$

where the c's are constants.

Analogous to the $l = 0$ case of Section 7 of I, the $l = 0$ case here can also be separated. Equation (6.2) now involves r only and becomes, together with (6.3),

$$\begin{aligned}\phi_{Sd}(r) = &-\frac{1}{360\pi}\left[\int_0^r dr'\,(f_0^2(r') - g_0^2(r'))r'^2(5r^4 + 10r^2r'^2 + r'^4)/r\right.\\ &\left.+\int_r^\infty dr'\,(f_0^2(r') - g_0^2(r'))r'(5r'^4 + 10r'^2r^2 + r^4)\right]\\ &+ d_{11}r + d_1 - d_{10}/r\end{aligned} \tag{6.8}$$

Equation (6.4) with $l = 0$ and $\phi_{Sd}(\mathbf{x}\to 0,\infty)$ replaced by (6.8) becomes

$$\left[\frac{1}{8}E_{0d}^3 + M_{bd}^3 + \phi_{Sd}(r) + \frac{1}{2}E_{0d}\Delta_0\right]g_0(r) - \left(\frac{1}{4}E_{0d}^2 + \Delta_0\right)\left(\partial_r + \frac{2}{r}\right)f_0(r) = 0 \tag{6.9a}$$

$$\left[\frac{1}{8}E_{0d}^3 - M_{bd}^3 - \phi_{Sd}(r) + \frac{1}{2}E_{0d}\Delta_1\right]f_0(r) + \left(\frac{1}{4}E_{0d}^2 + \Delta_1\right)\partial_r g_0(r) = 0 \tag{6.9b}$$

valid for all r. The amplitudes of the radial wave functions are limited by a conserved quantity derived in Appendix B. This quantity in (B3) is denoted by

$$N_{cd} = 2E_0^2 \int d^3\mathbf{x}\, j_{a\dot{a}} = 4 \int dr \cdot r^2\left(g_0^2(r) + \frac{1}{3} f_0^2(r)\right) \tag{6.9c}$$

Equations (6.8) and (6.9) form a one-dimensional singular nonlinear integrodifferential eigenvalue problem assigned to the $S = 1/2$, S-wave baryons.

Neglecting solutions diverging at the origin, (6.7) and (6.3) show that there are three possible solutions, which in the $r \to 0$ limit are of the form $c_{g0}(0)$, $c_{g0}(2)r^2$, and $c_{f0}(1)rY_{1m}$. These c's as well as $c_{d\infty}$ and τ_d are no longer free, but are determined by the nonlinear equations. The present model thus calls for a classification of the $S = 1/2$, S-wave baryons different from the nonrelativistic one in the literature (Lichtenberg, 1978, 1987; Particle Data Group, 1990), but similar to that of Appendix B of I. The lowest λ_d value in (6.7a) with $l = 0$ is tentatively assigned to the ground-state $J^p = (1/2)^+$ baryons.

Equations (6.8) and (6.9), indeed the general (6.4) with $\phi_{Sd}(\mathbf{x} \to 0, \infty)$ replaced by (6.2), can be solved exactly if the following approximations are made. Assume that a baryon is confined to a small region near the origin; (6.2) can be approximated by its last term. Assume further that in this state, the "kinetic energy" terms Δ_l and Δ_{l+1} are small relative to the squared masses E_{0d}^2; (6.4), now holding for all r, is simplified to

$$\left(-\frac{1}{8} E_{0d}^3 - M_{bd}^3 + d_{10}/r\right) g_0(r) + \left(\partial_r + \frac{l+2}{r}\right)\frac{1}{4} E_{0d}^2 f_0(r) = 0 \tag{6.10a}$$

$$\left(-\frac{1}{8} E_{0d}^3 + M_{bd}^3 - d_{10}/r\right) f_0(r) - \left(\partial_r - \frac{l}{r}\right)\frac{1}{4} E_{0d}^2 g_0(r) = 0 \tag{6.10b}$$

which has the same form as the radial equations derived from Dirac's equation for a hydrogen atom (Bethe and Salpeter, 1957) with the Coulomb interaction replaced by a scalar interaction. Thus, the solution of this classical problem can be simply modified for application to (6.10). The modification lies essentially in the indicial equation for (6.10). The eigenvalue of (6.10) can be expressed as

$$E_{0d} = {}_{(-)}^{+}\, 2M_{bd}[1 - 16d_{10}^2/E_{0d}^4(n_r + S_l + 1)^2]^{1/6} \tag{6.11a}$$

$$S_l + 1 = {}_{(-)}^{+}\, [(l+1)^2 + 16d_{10}^2/E_{0d}^4]^{1/2} \tag{6.11b}$$

where n_r is the radial quantum number. For $n_r = 0$, (6.11) is simplified to

$$E_{0d} = {}_{(-)}^{+}\, 2M_{bd}[1 + 16d_{10}^2/(l+1)^2 E_{0d}^4]^{-1/6} \tag{6.12}$$

For the ground state associated with (6.9), the above approximation leads to (6.12) with $l=0$. This approximation reduces the order of (6.4a) and (6.4b) by 2 and corresponds to keeping the $\lambda_d = l,\ -(l+1)$ solutions of (6.7a) only. The other solutions of (6.7) are removed by the approximation.

7. $S=3/2$ EQUATIONS

The $S=3/2$ baryon equations, (5.2) with totally symmetric χ in (2.6b) and ψ with ϕ_S replaced by ϕ_{Sq} of (5.6), are also generally not separable. In the $r\to 0$ and ∞ limits, however, separation is again possible. Analogous to the expansion in vector spherical harmonics in Appendix B of I, the χ's and ψ's defined in and below (2.6b) can be expanded as follows:

$$\chi_\mu(\mathbf{x}) = \sum_{\nu=-3/2}^{3/2} C(j+\nu, \tfrac{3}{2}, j; m-\mu, \mu) Y_{j+\nu\, m-\mu}(\vartheta, \varphi) g_{j+\nu}(r) \tag{7.1a}$$

$$\psi_\mu(\mathbf{x}) = \sum_{\nu=-3/2}^{3/2} (-)^{\nu+1/2} C(j+\nu, \tfrac{3}{2}, j; m-\mu, \mu) Y_{j+\nu\, m-\mu}(\vartheta, \varphi) g_{j+\nu}(r) \tag{7.1b}$$

where $\mu = \pm 3/2$ and $\pm 1/2$, and the C's are the usual Clebsch–Gordan coefficients.

At $r\to 0$ and ∞, (5.6), by analogy to (6.2) and (6.5), yields

$$\phi_{Sq}(\mathbf{x}\to 0) = -d_{30}/r \tag{7.2a}$$

$$\phi_{Sq}(\mathbf{x}\to\infty) = \frac{1}{192\pi} r^3 \int d^3\mathbf{x}\, \mathrm{Re} \sum_{\mu=-3/2}^{3/2} \chi_\mu(\mathbf{x}) \psi_\mu^*(\mathbf{x}) = \alpha_q^3 r^3 \tag{7.2b}$$

where d_{30} is a constant. The radial equations in these limits associated with the four spherical harmonics are given in Appendix C and hold for $j > 1/2$.

For $j = 1/2$, (7.1) and Appendix C no longer hold. Analogous to the case mentioned below (IB2), where the three kinds of spherical harmonics in the general vector spherical harmonics reduce to one for $l=0$, the four components in (7.1a) and (7.1b) can be shown to reduce to two components; the $h(r)$ and $k(r)$ terms in Appendix C drop out. Inserting the two-component form of (7.1a) and (7.1b) into (5.6) and making use of tables for the Clebsch–Gordan coefficients and normalized spherical harmonics one can reduce it to

$$\Delta\Delta\Delta\phi_{Sq}(r) = -\frac{1}{16\pi}(f^2(r) - g^2(r)) \tag{7.3}$$

independent of the angles.

Therefore, (5.2) as treated in Appendix C is separable for all r when $j=1/2$. Now, (C2c) and (C2d) also drop out, as the Clebsch–Gordan coefficients and spherical harmonics multiplying these can be shown to vanish for $j = 1/2$. The remaining (C2a) and (C2b), together with (7.3), by analogy to (5.5) and (6.8), become

$$\left[\frac{1}{8}E_{0q}^3 + M_{bq}^3 + \phi_{Sq}(r) + \frac{1}{2}E_{0q}\Delta_1\right]g(r) - \left(\frac{1}{4}E_{0q}^2 + \Delta_1\right)\left(\partial_r + \frac{3}{r}\right)f(r) = 0 \tag{7.4}$$

$$\left[\frac{1}{8}E_{0q}^2 - M_{bq}^3(r) - \phi_{Sq}(r) + \frac{1}{2}E_{0q}\Delta_2\right]f(r) + \left(\frac{1}{4}E_{0q}^2 + \Delta_2\right)\left(\partial_r - \frac{1}{r}\right)g(r) = 0$$

$$\begin{aligned}\phi_{Sq}(r) = \frac{1}{1920\pi}&\left[\int_0^r dr'\,(f^2(r') - g^2(r'))r'^2(5r^4 + 10r^2r'^2 + r'^4)/r\right.\\ &\left.+\int_r^\infty dr'\,(f^2(r') - g^2(r'))r'(5r'^4 + 10r'^2r^2 + r^4)\right]\\ &+ d_{31}r - d_{30}/r + d_3\end{aligned} \tag{7.5}$$

where the d's are integration constants. These equations now hold for all r and are assigned to the $S = 3/2$, S-wave baryons, together with a supplementary condition to be derived from (2.11) in a way similar to the way in which (6.9c) was.

These $S = 3/2$ equations are of the same form as the $S = 1/2$ equations (6.8) and (6.9). This resemblance is of the same nature as that between the $S = 0$, $l = 0$ equations (I7.3) and (I7.4) and the $S = 1$, $l = 0$ equations (I8.3) and (I8.4).

For large r, (7.5) can be put in the form

$$\phi_{Sq}(r \to \infty) = -\alpha_q^3 r^3 \tag{7.6}$$

and (7.4) yields the asymptotic form

$$f(r \to \infty) = q(r \to \infty) = c_{g\infty} r^{\tau_q} e^{-\alpha_q r^2/2} \tag{7.7}$$

which is confining for $\alpha_q > 0$. Otherwise, f or g in (7.7) has to change sign. Here, $c_{g\infty}$ and τ_q are also not free constants, but are fixed by the nonlinear equations. In the $r \to 0$ limit, (6.7), dropping the subscript 0, with $l = 1$ applies. Radial solutions converging at the origin are then of the form $c_g(1)r$, $c_g(3)r^3$, and $c_f(2)r^2$. Analogous to the $S = 1/2$ case, the $c_g(1)r$ solution is tentatively assigned to the ground-state $S = 3/2$ baryons.

Equations (7.4) and (7.5) can also be solved exactly if two approximations like those assumed above (6.10) are made. In this case, (6.11) also applies if $l = 1$, the subscript d is replaced by q, and $d_{10} \to d_{30}$. The equivalent of (6.12) becomes

$$E_{0q} = {}_{(-)}^{\;+}\, 2M_{bq}[1 + 4d_{30}{}^2/E_{0q}{}^4]^{-1/6} \tag{7.8}$$

associated with the $c_g(1)r$ solution near the origin.

8. MODEL FOR INTERNAL FUNCTION AND MASS OPERATOR

To obtain M_b^3 from (3.8), the internal functions $\xi^{\{ps\}q}$ and mass operator $m_{3\text{op}}$ are considered below. The approach is entirely similar to that of Section 9 of I.

Assume, as in I, that

$$\xi_A^p(z_{\text{I}}) \to z_{\text{I}}^p, \qquad \xi_C^s(z_{\text{III}}) \to z_{\text{III}}^s, \qquad \xi_B^q(z_{\text{II}}) = z_{\text{II}}^q \tag{8.1}$$

The left sides, in order, have been associated with the three quark equations (2.1), (2.3), and (2.2), respectively. Analogous to Section 9 of I, the association is also possible if z_{I}, z_{II} and z_{III} in (8.1) are permuted. For example, (2.1) can be associated with any one of the three internal functions; $\chi_{A\dot{B}}(\text{I})\xi_A^p(z_{\text{I}})$, $\chi_{A\dot{B}}(\text{I})\xi_A^p(z_{\text{II}})$, and $\chi_{A\dot{B}}(\text{I})\xi_A^p(z_{\text{III}})$ are all possible. Thus, the three internal coordinates are new degrees of freedom similar to that of color in QCD. The so-called R ratio$=\sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$ in e^+e^- annihilation may in principle not be affected if the color degrees of freedom are replaced by those of the z's.

In (I9.1), there are two linear combinations. The analogous specialization of (3.1) into products of the right sides in (8.1) leads to six linear combinations of terms consisting of I, II, and III permutations of $z_{\text{I}}^p z_{\text{II}}^s z_{\text{III}}^q$. These six combinations reduce to three under the restriction of $z_{\text{III}} \to z_{\text{I}}$ of (3.3). With the notations $z_{\text{I}} \equiv z$ and $z_{\text{II}} \equiv u$ in Section 9 of I, (3.4) is thus specialized to

$$\eta_r^p(z_{\text{I}}, z_{\text{II}}) \to z^p z^s u^q \varepsilon_{rsq} \tag{8.2a}$$

$$\xi^{\{psq\}}(z_{\text{I}}, z_{\text{II}}) \to z^p z^s u^q + z^p u^s z^q + u^p z^s z^q \tag{8.2b}$$

These forms are the same as those in the literature (Lichtenberg, 1978) if two of the quarks in the latter have the same coordinates.

The $m_{3\text{op}}$ operators in the middle of (3.5) are of the form (I9.3a). By analogy to (I9.5), the general form of the right side of (3.5) is

$$m_{3\text{op}} = m_{3\text{op}}(z^v, u^v, \partial_{zv} + \partial_{uv}, m_v)_{v=p,s,q} \tag{8.3}$$

Further specialization of (8.3) depends upon the forms of the baryon internal functions, analogous to the assumed forms of (I9.6a) and (I9.8a). Consider first the symmetric (8.2b). The simplest form of (8.3) yielding a dimension of cubed mass, employing the associations of $m_s\partial_{zs}$ and $m_s\partial_{us}$ below (I9.5), is

$$m_{3\text{op}}(p, s, q)_S = \left[\sum_{v=p,s,q} \frac{1}{2} m_v(z^v\partial_{zv} + u^v\partial_{uv})\right]^3 \tag{8.4}$$

which is simply the cube of half of the bare quark mass summing operator.

When applied to (8.2b), it yields the eigenvalue

$$M_b^3(p, s, q)_S = \frac{1}{8}(m_p + m_s + m_q)^3 \tag{8.5}$$

For the mixed symmetry (8.2a), the form of (8.4) is neither necessary nor sufficient. Consider first $p \neq r$; there will be one repeated index on the right side of (8.2a). Let the indices be p, p, and q. One of the forms next to (8.4) in simplicity is to include terms like those of (I9.6a):

$$m_{3\text{op}}(p, p, q)_M = \left[\frac{1}{2}m_p(z^p\partial_{zp} + u^p\partial_{up}) + \frac{1}{2}m_q(z^q\partial_{zq} + u^q\partial_{uq})\right]^3 \tag{8.6}$$

where the bracketed terms are again half the bare quark mass summing operator. This internal operator is symmetric in z and u as well as in p and q in spite of the fact that the left side of (3.5) is not. However, this is in agreement with the corresponding space-time generalization (2.8), where the left side is not symmetric in I and II, but the right side, by (2.10) and (5.3), is. Application of (8.6) to (8.2a) yields the eigenvalue

$$M_b^3(p, p, q)_M = \frac{1}{8}(2m_p + m_q)^3 \tag{8.7}$$

the Σ^0 and Λ^0 internal functions correspond to those of π^0 of (I9.7), with the lower sign and $u^p \to -u^p$, and η of (I9.9a), respectively. By (8.2a), these have the internal functions

$$\eta_1^1 - \eta_2^2 \to 2z^1z^2u^3 - z^3(z^2u^1 + z^1u^2), \qquad \Sigma^0 \tag{8.8a}$$

$$\eta_1^1 + \eta_2^2 - 2\eta_3^3 \to 3z^3(z^2u^1 - z^1u^2), \qquad \Lambda^0 \tag{8.8b}$$

which involve all three indices. Therefore, (8.6) has to be generalized to include a third index to become

$$m_{3\text{op}}(p, s, q)_M = m_{3\text{op}}(p, s, q)_S \tag{8.9}$$

Equation (8.8) exhibits additional symmetry under the interchange of the indices 1 and 2. Following the discussion below (I9.7) for the analogous meson cases, we modify $m_{3\text{op}}$ of (8.9) further to include similar index symmetrizing operators corresponding to the bracketed expression in (I9.8a). Among the operator terms of (8.9), the only one that allows such a symmetrizing and has (8.8) as eigenfunctions with nonvanishing eigenvalues is

$$m_{3\text{op}}^{123} = \frac{3}{4}m_1m_2m_3(z^3\partial_{z3} + u^3\partial_{u3})I^{(12)} \tag{8.10a}$$

$$I^{(12)} = z^1z^2\partial_{z1}\partial_{z2} + u^1u^2\partial_{u1}\partial_{u2} + z^1u^2\partial_{z1}\partial_{u2} + u^1z^2\partial_{u1}\partial_{z2} \tag{8.10b}$$

Under the interchange of the upper indices, (8.10b) becomes

$$I^{(21)} = z^2 z^1 \partial_{z1} \partial_{z2} + u^2 u^1 \partial_{u1} \partial_{u2} + z^2 u^1 \partial_{z1} \partial_{u2} + u^2 z^1 \partial_{u1} \partial_{z2} \tag{8.10c}$$

and (8.8) remains as eigenfunctions of (8.10c). By analogy to the construction of (I9.8a), $I^{(12)}$ of (8.10a) is now replaced by the index-symmetrized form

$$I^{\{12\}} = \frac{1}{2}(I^{(12)} + I^{(21)}) \tag{8.10d}$$

and (8.10a) is replaced by

$$m_{3\text{op}}^{\{12\}3} = \frac{6}{8} m_1 m_2 m_3 (z^3 \partial_{z3} + u^3 \partial_{u3}) I^{\{12\}} \tag{8.10e}$$

For Σ^0 and Λ^0, therefore, (8.6) is replaced by

$$m_{3\text{op}}(p, s, q)_{M\{12\}} = m_{3\text{op}}(p, s, q)_M - m_{3\text{op}}^{123} + m_{3\text{op}}^{\{12\}3} \tag{8.11a}$$

Application to (8.8) yields the eigenvalues

$$M_b^3(\Sigma^0) = \frac{1}{8}(m_1 + m_2 + m_3)^3 \tag{8.11b}$$

$$M_b^3(\Lambda^0) = M_b^3(\Sigma^0) - \frac{3}{4} m_1 m_2 m_3 \tag{8.11c}$$

The index-symmetrized operator (8.10d) plays a role similar to that of I^2, the square of the isospin operator, and differentiates Λ^0 with $I^2 = 0$ from Σ^0 with $I^2 = 2$. There is also a corresponding index-symmetrizing operator for mesons, which, when operating upon the ω and ρ^0 internal functions (I9.7), produces zero eigenvalues.

For charmed baryons, the above SU_3 results can be taken over if one of the indices 1, 2, and 3 is replaced by the index 4. Thus, the eigenvalues for Λ^0, Σ^0, and Σ^+ and Σ^- of (8.11b), (8.11c), and (8.7) become those for Λ_c^+, Σ_c^+, and Σ_c^{++} and Σ_c^0, respectively, if the index 3 is replaced by 4 (Lichtenberg, 1978). The corresponding eigenfunctions of and eigenvalues for Ξ^{A+} and Ξ_c^{A0} are simply those for Λ_c^+ with its index $2 \to 3$ and index $1 \to 3$, respectively.

9. APPLICATION AND DISCUSSION

In this section, application of the present formalism is given on an overall level and shown to be capable of accounting for several basic observations [see point (a) below] not accountable by QCD. Comparison of data to predictions of analytical but simplified, hence approximative, solutions [see point (b)] shows acceptable agreement. The full numerical integration program required to check the theory by data is beyond the scope of this paper, but is discussed together with classification implications under point (c).

(a) The harmonic type of confinement shown in (6.6) and (7.7) arises from the same basic formalism that led to linear confinement for mesons in (I7.6a). The radial equations (6.8) and (6.9) for ground-state, spin-1/2 baryons are specialized from the basic covariant (2.9) and (2.10) without approximation except for the assumption of zero relative energy ω_0 in (B1). This assumption can perhaps be removed in a quantized treatment, as has been shown for the meson case (Hoh, 1994). The same basically holds for the ground-state spin-3/2 baryon equations (7.4) and (7.5) supplemented by a relation corresponding to (6.9c). Therefore, numerical solutions of these equations will include full relativistic effects.

Scalar and pseudoscalar interactions as basic interactions of nature on a par with the vector or electromagnetic interaction also find natural places in the present formalism.

(b) At this level, the ground-state, $S = 1/2$ and $S = 3/2$ baryons are treated making use of the approximate results of (6.12) and (7.8) together with the M_b^3 formulas of Section 8, neglecting the u–d-quark mass difference. For nucleons, $p, q = 1, 2$, and (8.7) becomes

$$M_b(N) \approx \frac{1}{2}(2m_1 + m_2) \approx M_b(\Delta), \qquad m_1 = m_2 \tag{9.1}$$

which is the same as the mass $M_b(\Delta)$ obtained from (8.5). Putting $E_{0\mathrm{d}}$ and $E_{0\mathrm{q}}$ equal to the measured neutron and Δ^0 masses (Particle Data Group, 1990) and $l = 0$ in (6.12) and eliminating $M_{b\mathrm{d}} = M_{b\mathrm{q}}$ by means of (7.8) yields

$$d_{10}^2 - 0.431 d_{30}^2 \approx 0.2 \tag{9.2}$$

To be consistent with $d_{m0} = d_{m1}$ below (I10.2), the d's here are also assumed to be equal and (9.2) yields $d_{30} = d_{10} = 0.593\ \mathrm{GeV}^2$. With this value, (7.8) and (8.5) yield the bare quark mass $m_1 = m_2 = 0.445$ GeV (same units as used below), in close agreement with the corresponding upper limit values of 0.437 and 0.447 estimated from the π and ω data below (I10.2).

The bracketed expression in (7.8) and particularly in (6.12) deviates appreciably from 1 for the lighter baryons, so that the "equal-spacing" rule indicated by (8.5) is somewhat modified. Making use of the Δ^-, Σ^{*-}, Ξ^{*-}, and Ω^- masses for $E_{0\mathrm{q}}$, we find that (7.8) and (8.5) yield the approximate equal spacing of $m_3 - m_2 = 0.125$, 0.130, and 0.124, which has a spread about half that of E_{0q}. These yield $m_3 = 0.572$, 0.577, and 0.571, respectively, which are somewhat higher than the upper limit values 0.559 and 0.554 obtained from the K and φ data below (I10.2).

With the known neutron, Σ^0, and Ξ^0 masses, application of (6.12) with $l = 0$, (9.1), (8.11b), and (8.7) yields the unequal spacings $m_3 - m_2 = 0.154$ and 0.08. These lead to $m_3 = 0.599$ and 0.525, yielding a mean value of 0.562, comparable to the m_3 values above. The Λ^0 mass obtained from (8.11c) and (6.12) becomes 1.027, much less than the measured 1.116.

The same procedure can be taken over for the charmed baryons according to the end of Section 8. The Σ_c mass of 2.453 yields $m_4 = 1.623$, not far from the upper limit values 1.62, 1.591, and 1.563 estimated from the D, D_s and J/ψ data below (I10.2). With this m_4 value, a Λ_c^+ mass of 2.3386 is obtained, which exceeds the measured 2.285. The Ξ_c^{A+} and Ξ_c^{A0} masses are obtainable from (8.11c) with index 1 or 2 replaced by 4.

(c) As is shown by (6.7), there are three regular solutions near $r = 0$. Similar to the assignments near the end of Section 10 of I, a particle classification different from that according to nonrelativistic quantum mechanics is called for. For example, assuming that eigensolutions exist, the nucleon is naturally assigned to $\lambda_d = l$, $l = 0$ in (6.7). The corresponding eigensolutions associated with $\lambda_d = l + 1$ and $l + 2$, $l = 0$, in (6.7) refer to the same nucleons, not different particles. Compared to the nucleon wave functions, (6.7) shows that these wave functions extend further out radially and can possibly be associated with higher energies, as is indicated by the form of (6.8).

The baryon equations are more complex than the meson equations of I. In addition to being of higher order, E_0 and M_b do not appear together as a single eigenvalue as they do in I. Unlike the meson equations (I7.3) and (I8.3), (6.9) and (7.4) show that the radial wave functions depend upon the quark content via M_b. By (6.5b), (6.6), (7.6), and (7.7), the strong interaction radii of the baryons depend upon their quark content, contrary to the meson case mentioned below (I10.1).

For angularly excited states $l \neq 0$, separation in the relative r and ϑ space shown in (6.3) and (7.1) no longer holds. Thus (5.4), (5.5), (5.2) and (5.6) together with relations corresponding to (6.9c) need to be integrated in two dimensions. The numerical program for this nonseparable case is obviously of a higher order of magnitude.

Possible nonseparable solutions may be classified differently from the usual separable ones employed in literature (Lichtenberg, 1987; Particle Data Group, 1990). The same holds for angularly excited meson states governed by (I6.7), (I6.8), (I7.2), and (I8.1). Since nonseparability couples the r and ϑ degrees of freedom, it may allow fewer solutions than does the separable case, in which the number of solutions is essentially the number of angular solutions times the number of radial solutions. If so, this may account for the large number of unobserved states predicted in the literature.

APPENDIX A. NEGLECTED TERMS IN (2.7) AND (2.9)

Consider first the transition from (2.7) to (2.10). The product $\chi_{B\dot{b}}(\mathrm{I})\psi^{h}_{B}(\mathrm{I})\chi_{A\dot{a}}(\mathrm{II})$ in the fourth term in the first pair of braces of (2.7) differs from the middle expression of (2.5a) basically in that the dotted indices are not symmetric, as they refer to different coordinates. Therefore, a generalization according to (2.5a) will not lead to a reduction from eight to six components. Similarly, the corresponding product $\chi_{B\dot{b}}(\mathrm{I})\psi^{a}_{B}(\mathrm{I})\psi^{h}_{A}(\mathrm{II})$ in the third term there will also involve eight components after generalization, in contrast to the six components of $\psi^{\{ag\}}{}_{\dot{e}}(\mathrm{I}, \mathrm{II})$ above (2.6a).

An analogous generalization of the product $\chi_{B\dot{b}}(\mathrm{I})\chi_{B\dot{h}}(\mathrm{I})\chi_{A\dot{a}}(\mathrm{II})$ in the second term there will make it totally symmetric owing to (2.5) and hence leads to four components only. Thus, these three product wave functions, when generalized, cannot represent the baryons in a unique way and are therefore put to zero. Only the product $\chi_{B\dot{b}}(\mathrm{I})\chi_{B\dot{h}}(\mathrm{I})\psi^{a}_{A}(\mathrm{II})$ in the first term there survives by virtue of (2.5). The corresponding terms in the second pair of braces and the c.c. terms follow suit and (2.10) results.

Turning to the transition from the products of (2.1)–(2.3) to (2.9), the above results also apply. In addition, by the same resoning preceding (I4.11), there are no free quarks in the baryon system, so that the ψ^{a}'s and $\chi_{\dot{b}}$'s in the product equations, generalized according to (2.5), as well as (2.1)–(2.3) themselves now drop out. By (2.4), the V's also vanish, in agreement with the nonobservation of massless scalar particles. Therefore, so do the mmV terms on the right sides of the product equations. The products $V(\mathrm{I})V(\mathrm{I})$ and $V(\mathrm{I})V(\mathrm{II})$ can be obtained by multiplying together two of (2.4a)–(2.4c). After generalizations analogous to (2.5), the right sides of the equations will contain functions of diquark type ψ^{ac}, $\chi_{\dot{b}\dot{d}}$ and meson type $\psi^{a}_{\dot{c}}$, $\chi^{b}_{\dot{d}}$. As free diquarks and mesons are apparently absent in pure baryon systems, the corresponding wave functions of these types can be put to zero. Hence the mVV terms also drop out. The remaining VVV terms are generalized according to (2.8), and (2.9) is obtained.

APPENDIX B. CONSERVED QUANTITY FOR $S=1/2$ AND $l=0$

With the same set of specializations in Sections 5 and 6 that convert (2.9) and (2.10) into (6.8) and (6.9), S_{ad} of (2.12b) can be written as a divergence in the relative x_1 and x_2 directions. Therefore, $\int S_{ad}\,d\mathbf{x}^3=0$.

Repeating the same procedure for (2.12a) leads to

$$\int D_{ad}\,d^3\mathbf{x}=\left[\left(\frac{3}{2}+\frac{\omega_0}{E_0}\right)\partial_{X^0}-\partial_0\right]\frac{2}{3}E_0^2\int d^2\mathbf{x}\,j_{a\dot{a}}$$
$$-2\left(\frac{\omega_0}{E_0}\partial_{X^0}+\partial_0\right)\frac{16}{9}\int d^3\mathbf{x}\,(\chi_{0a}(\partial_1^2+\partial_2^2)\chi_{0\dot{a}}+\psi_0^{\dot{a}}(\partial_1^2+\partial_2^2)\psi_0^a)$$
$$+\text{imaginary terms}=0 \tag{B1}$$

where

$$j_{a\dot{b}}=\chi_{0a}(\mathbf{x})\chi_{0\dot{b}}(\mathbf{x})+\psi_{0a}(\mathbf{x})\psi_{0\dot{b}}(\mathbf{x}) \tag{B2}$$

Here, χ_{0a} and $\chi_{o\dot{b}}$ are the complex conjugates of $\chi_{0\dot{a}}$ and ψ_{0b}, respectively, and ∂_1 and ∂_2 refer to derivations in the relative x_1 and x_2 directions, respectively. The integrals in (B1) are independent of the relative time x^0, so that the ∂_0 terms make no contribution.

Below (9.6) of (Hoh, 1994) it was shown that the relative energy ω_0 associated with the quarks in a meson vanishes in the rest frame, as a result of a quantized treatment. This lends support to the assumption, made in the absence of such a quantized treatment here, that the relative energy ω_0 in (B1) is also zero. The second term on the right of (B1) therefore vanishes.

While the first two terms on the right of (B1) are real, the last one is imaginary. Addition of (2.11a) and (2.11b) therefore removes this imaginary term. Integration of this sum over $\mathbf{x}$ thus yields

$$\partial_{X^0}2E_0^2\int d^3\mathbf{x}\,j_{a\dot{a}}=0 \tag{B3}$$

Here, the integral is a conserved quantity independent of the laboratory time X^0, analogous to the integrals in (A3) of Hoh (1994). The $j_{a\dot{b}}$ and $j_{a\dot{a}}$ become the probability current density and probability density, respectively, if (2.9) is replaced by a Dirac equation, as was mentioned at the end of Section 2.

APPENDIX C. RADIAL EQUATIONS FOR $S=3/2$ AT $r\to 0$ AND ∞

Substitute (7.1) and (2.6b) and a similar relation for the ψ's below (2.6b) into (5.2) and put the doublets $\chi_{0\dot{a}}$ of (2.6a) and ψ_0^a to zero. Let further $\phi_S(\mathbf{x})$ there be replaced by (7.2), $E_0 \to E_{0q}$, and $M_b^3 \to M_{bq}^3$, where q signifies association with $S = 3/2$. The eight component equations of (5.2) can be shown, after considerable algebra making use of tables of Clebsch–Gordan coefficients, to degenerate into the same set of four radial equations (C2a)–(C2d) below, associated with the four spherical harmonics $Y_{j+\nu,\nu} = 3/2,\ 1/2,\ -1/2$, and $-3/2$, respectively. Let

$$g_{j+3/2} = if, \qquad g_{j+1/2} = g, \qquad g_{j-1/2} = ih, \qquad g_{j-3/2} = k \tag{C1}$$

These equations read

$$\begin{aligned}
&\left[\frac{1}{8} E_{0q}^3 - M_{bq}^3 - \phi_{Sq}(\mathbf{x} \to {}^{0}_{\infty}) + \frac{1}{2} E_{0q} \frac{3}{2j+2} \Delta_{j+3/2}\right] f(r) \\
&\quad - \frac{1}{2} E_{0q} \frac{[3(2j-1)(2j+3)]^{1/2}}{2j+2} \Delta_{(j-1/2)-} h(r) \\
&\quad + \frac{1}{4} E_{0q}^2 \left(\frac{3j}{j+1}\right)^{1/2} \left(\partial_r + \frac{j+1/2}{r}\right) g(r) \\
&\quad + \left(\frac{3}{4j(j+1)}\right)^{1/2} \Delta_{j+3/2} \left(\partial_r - \frac{j+1/2}{r}\right) g(r) \\
&\quad - \left(\frac{(2j-1)(2j+3)}{4j(j+1)}\right)^{1/2} \Delta_{(j-1/2)-} \left(\partial_r - \frac{j-3/2}{r}\right) k(r) = 0
\end{aligned} \tag{C2a}$$

$$\begin{aligned}
&\left[-\frac{1}{8} E_{0q}^3 - M_{bq}^3 - \phi_{Sq}(\mathbf{x} \to {}^{0}_{\infty}) + \frac{1}{2} E_{0q} \frac{4j-3}{2j} \Delta_{j+1/2}\right] g(r) \\
&\quad + \left[\frac{1}{4} E_{0q}^2 \left(\frac{3j}{j+1}\right)^{1/2} \left(\partial_r + \frac{j+5/2}{r}\right)\right. \\
&\quad \left. + \left(\frac{3}{4(j+1)}\right)^{1/2} \Delta_{j+1/2} \left(\partial_r + \frac{j+5/2}{r}\right)\right] f(r) \\
&\quad + \left[\frac{1}{4} E_{0q}^2 \left(\frac{(2j-1)(2j+3)}{j(j+1)}\right)^{1/2} \left(\partial_r - \frac{j-1/2}{r}\right)\right. \\
&\quad \left. - \left(\frac{(2j-1)(2j+3)}{4j(j+1)}\right)^{1/2} \Delta_{j+1/2} \left(\partial_r - \frac{j-1/2}{r}\right)\right] h(r) \\
&\quad + \frac{1}{2} E_{0q} \frac{1}{2j} [3(2j-1)(2j+3)]^{1/2} \Delta_{(j-1/2)-} k(r) = 0
\end{aligned} \tag{C2b}$$

$$\left[\frac{1}{8}E_{0q}^3 - M_{bq}^3 - \phi_{Sq}(\mathbf{x}\to{}^0_\infty) - \frac{1}{2}E_{0q}\frac{4j+7}{2j+2}\Delta_{j-1/2}\right]h(r)$$

$$+\left[\frac{1}{4}E_{0q}^2\left(\frac{(2j-1)(2j+3)}{j(j+1)}\right)^{1/2}\left(\partial_r+\frac{j+3/2}{r}\right)\right.$$

$$\left.-\left(\frac{(2j-1)(2j+3)}{4j(j+1)}\right)^{1/2}\Delta_{j-1/2}\left(\partial_r+\frac{j+3/2}{r}\right)\right]g(r)$$

$$+\left[\frac{1}{4}E_{0q}^2\left(\frac{3(j+1)}{j}\right)^{1/2}\left(\partial_r-\frac{j-3/2}{r}\right)\right.$$

$$\left.-\left(\frac{3}{4j(j+1)}\right)^{1/2}\Delta_{j-1/2}\left(\partial_r-\frac{j-3/2}{r}\right)\right]k(r)$$

$$-\frac{1}{2}E_{0q}\frac{1}{2j+2}[3(2j-1)(2j+3)]^{1/2}\Delta_{(j+3/2)+}f(r)=0 \qquad \text{(C2c)}$$

$$\left[-\frac{1}{8}E_{0q}^3 - M_{bq}^3 - \phi_{Sq}(\mathbf{x}\to{}^0_\infty) + \frac{1}{2}E_{0q}\frac{3}{2j}\Delta_{j-3/2}\right]k(r)$$

$$+\left[\frac{1}{4}E_{0q}^2\left(\frac{3(j+1)}{j}\right)^{1/2}\left(\partial_r+\frac{j+1/2}{r}\right)\right.$$

$$\left.-\left(\frac{3}{4j(j+1)}\right)^{1/2}\Delta_{j-3/2}\left(\partial_r+\frac{j+1/2}{r}\right)\right]h(r)$$

$$-\left(\frac{(2j-1)(2j+3)}{4j(j+1)}\right)^{1/2}\Delta_{(j+1/2)+}\left(\partial_r+\frac{j+5/2}{r}\right)f(r)$$

$$+\frac{1}{2}E_{0q}\frac{1}{2j}[3(2j-1)(2j+3)]^{1/2}\Delta_{(j+1/2)+}g(r)=0 \qquad \text{(C2d)}$$

where

$$\Delta_{j+}=\left(\partial_r+\frac{j}{r}\right)\left(\partial_r+\frac{j+1}{r}\right), \qquad \Delta_{j-}=\left(\partial_r-\frac{j+1}{r}\right)\left(\partial_r-\frac{j}{r}\right) \qquad \text{(C3)}$$

These equations hold for $j>1/2$ only.

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